Sub Shot-Noise Phase Sensitivity with a Bose-Einstein Condensate Mach-Zehnder Interferometer

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Quantum-enhancement measurement techniques are the subject of lively research in modern technology. Bose Einstein Condensates (BECs), with their coherence properties, have attracted wide interest for their possible application on ultra precise interferometry and ultra weak force sensors. Since BECs, unlike photons, are interacting, they may permit the realization of the specific quantum states needed as input of an interferometer to approach sensitivities surpassing the classical shot noise limit $1/N^{1/2}$ (being N the total number of particles). In particular, several efforts focus on the possibility of interferometrically estimated phases with precision bounded by the Heisenberg limit $1/N$, which is widely believed to be the ultimate limit imposed by quantum mechanics.

We have studied the sensitivity of a matter-wave Mach-Zehnder interferometer to an external weak phase perturbation, see Fig. 1. The inputs are two Bose-Einstein condensates created by splitting a single condensate in two parts [1]. The recent experimental creation of very stable double-well traps [2, 3] bodes well for the future of matter-wave interferometry. The interferometric phase sensitivity depends on the specific quantum state created with the two condensates. This quantum state can be taylored by varying the time scale τ of the splitting process and the tunneling coupling energy $K(t)$ (which is proportional to the overlap of the wavefunctions localized in each well). We have studied the dynamical splitting of the two wells solving the bosonic quantum field equation in two-mode approximation. The MZ phase uncertainty, $\Delta\theta$, as a function of the splitting time and the tunneling coupling were calculated using the error propagation formula

$$(\Delta\theta)^2 \approx \frac{(\Delta \hat{J}_z^{out})^2}{|\partial\langle \hat{J}_z^{out}\rangle/\partial\theta|^2},$$

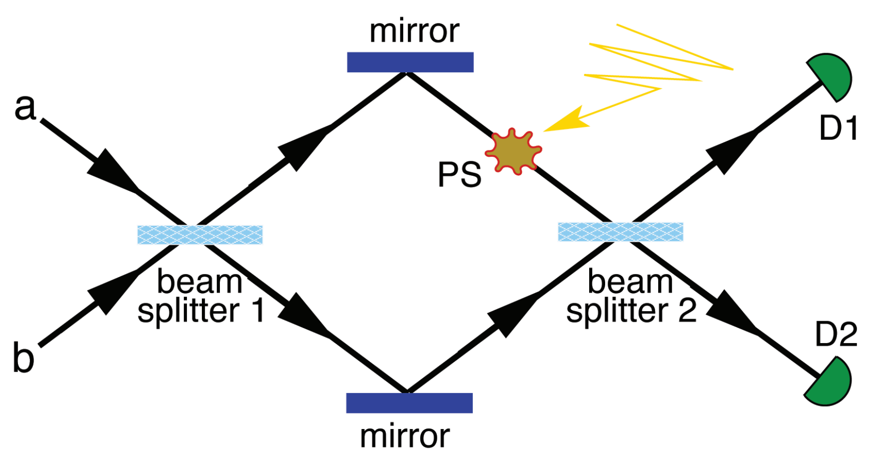

Fig. 1. Schematic representation of the Mach-Zehnder interferometer. Atoms/photons enter the a and b input ports, mix and recombine in the beam splitters and are finally detected in D1 and D2. The phase shift is inferred from the number of atoms/photons measured in each output port.

with $\hat{J}^{out}$ the relative number of particles operator whose expectation values are calculated at the output ports.

We found three different regimes, see Fig. 2, which, in analogy to three corresponding regimes existing in the dynamical Josephson effect, we termed Rabi, Josephson, and Fock. These three regimes are characterized by different scalings of the phase sensitivity $\Delta\theta$ with the total number of condensate particles N.

Rabi Regime. This is a semiclassical regime characterized by a strong tunneling coupling K between the two condensates. In this regime the phase sensitivity scales at the standard quantum limit $\Delta\theta \sim /N^{1/2}$.

Josephson Regime. This is the most interesting regime since it can be reached in a realistic experimental setting, see Fig. 3 (ramping time $\Delta t_R \approx 100$ msec and final distance between the wells $d_R \approx 10$ μm) [2, 3]. By creating the states feeding the interferometer in this regime, it is possible to reach a sub shot-noise sensitivity $\Delta\theta \sim 1/N^{3/4}$. We have also found that the $1/N^{3/4}$ scaling is a rigorous upper bound in the limit $N \to \infty$, while keeping constant all different parameters of the bosonic Mach-Zehnder interferometer.

Fock Regime. In this regime, the initial condensate has been fragmented into two independent condensates. Once these feed the input ports of the MZ, it is possible to reach the Heisenberg limit $\Delta\theta \sim 1/N$. However, in a realistic dynamical BEC splitting, the $1/N$ limit requires a very long adiabaticity time-scale, which is hardly achievable experimentally.

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[1] L. Pezzé, et al., *Phys. Rev. A.* **72**, 043612 (2005).
[2] Y. Shin, et al., *Phys. Rev. Lett.* **92**, 050405 (2004).
[3] M. Albiez, *Phys. Rev. Lett.* **95**, 010402 (2005).

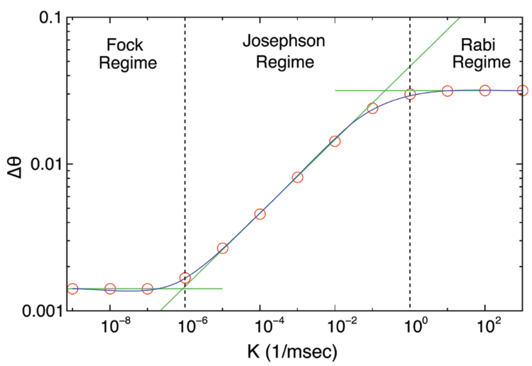

Fig. 2.
Mach Zehnder phase sensitivity $\Delta\theta$ as given by Eq. 1, as a function of the tunneling coupling energy K. The green lines are the analytical predictions in three regimes: i) Rabi Regime, where $\Delta\theta \approx 1/\sqrt{N}$; ii) Josephson Regime, where $\Delta\theta \approx 1/N^{3/4}$; and iii) Fock Regime, where $\Delta\theta \approx 1/N$. Here N = 1000. The red circles are given by exact numerical solutions, while the blue line is an approximate, variational, calculation.

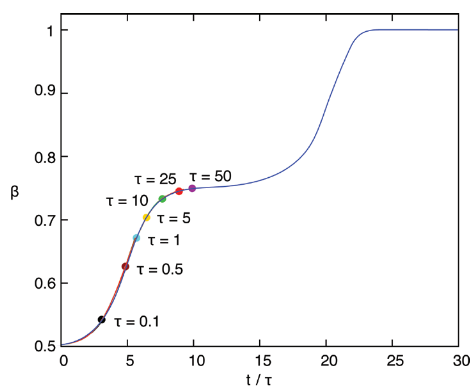

Fig. 3.
Plot of the scaling parameter β defined as $\Delta\theta = \alpha/N^{\beta}$, as a function of time. The calculation has been made with N = 1000 and N = 10000, particles. The blue line represents the adiabatic behavior; the points correspond to the minimum of MZ phase sensitivity occurring for different values of the effective ramping time τ (in msec),

$$\tau = \frac{\Delta t_{ramp}}{d_R\sqrt{V_0 - \mu}}$$

with Δt_{ramp} the ramping time, d_R the final distance between the wells, V_0 the height of the potential barrier at the end of the splitting, and μ the chemical potential.